THE ASTROPHYSICAL JOURNAL, 313:L75–L79, 1987 February 15

OVERTONE PULSATION IN STARS

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Received 1986 October 8; accepted 1986 November 20

ABSTRACT

It is well known that type "c" RR Lyrae stars pulsate in the first-overtone mode. These stars also show a distinctive light curve shape. We show, using a simple model, that this shape is indeed an expected feature of the overtone and is caused by the reversal of phase of the luminosity variation at the pulsation node, deep in the star. This result also applies to other types of radially pulsating stars and should aid in identifying overtone pulsation directly from the light curve shape. The model is then used to predict the light variation of type "e" (second-overtone) pulsators.

Subject headings: stars: pulsation — stars: RR Lyrae

I. PULSATION MODES AND MODELS

In 1940 Schwarzschild analyzed the periods of RR Lyrae stars in M3 and showed that a certain subset of stars (designated "type c" by Bailey; 1899, 1902*a*, *b*), pulsate in the first-overtone mode, while the remainder (types "a" and "b") are fundamental pulsators. The light curves of the type c stars are more sinusoidal than the others, but it has never been very clear whether this is due to the different mode, or some other property of the pulsation, such as the amplitude, period, or effective temperature. Schwarzschild's argument involved the distribution of periods of stars in a cluster and does not help to determine the mode of pulsation of any individual field RR Lyrae star or other type of star. It is therefore of interest to know the direct effects of mode of pulsation on the light variation.

Figure 1 is adopted from Ledoux and Walraven (1958), which is based on the observations of the RR Lyraes in Omega Centauri discussed by Martin (1938). Types a and b show a rapid rise in luminosity followed by a more gradual decline, while the type c light curve tends to be sinusoidal in shape and often shows a distinct bump near maximum light. Amplitudes of the type c stars are typically 0.5 mag, those of the type b stars range from 0.5 to about 1.0, and those of the type a stars from about 1.0 to 1.5 mag (photographic). As a function of period there is a sharp break between the type c and type a stars, but a gradual change from type a to type b stars with increasing period and decreasing amplitude. The distinctness of the type c curve is confirmed by detailed analysis of the light curve shape using Fourier analysis (Simon and Teays 1982; Petersen 1984; Stellingwerf and Dickens 1987). The trends in RR Lyrae light curves can be seen in the analysis of NGC 6171 by Dickens (1970).

Detailed hydrodynamic models of fundamental and overtone pulsations can reproduce the observed light curves (cf. Stellingwerf 1975). These models suffer from several difficulties when dealing with overtone pulsations. The effect of coarse zoning near the photosphere is much more noticeable than in fundamental models and can cause large distortions of the light variation. These models are complicated, and it is often impossible to disentangle the many physical processes and numerical effects that contribute to the final result.

It was with a view toward understanding the processes that drive the pulsation that Baker (1966) introduced the one-zone pulsation model. The model proved very useful in this regard, clearly showing the interplay of thermodynamics, geometry, and opacity in an unstable stellar envelope. It was subsequently shown (Stellingwerf 1972, hereafter S72) that by restricting the single zone to a thin shell near the stellar surface, the nonlinear one-zone model portrayed an actual pulsating star quite well. In fact, if the luminosity variation at the base of the shell is taken into account, a very realistic light variation is obtained. Fourier analysis confirms that the one-zone light curves match those of observed RR Lyrae type a stars (Stellingwerf and Donohoe 1986, 1987).

The one-zone model is successful because the amplitude of pulsation of a giant star (such as an RR Lyrae star or a classical Cepheid) decreases rapidly as one proceeds from the surface toward the interior of the star. An actual star resembles in many respects a model consisting of a uniform pulsating layer resting on a solid, fixed core.

We now introduce a new idea: this model should be even more appropriate in the case of an overtone pulsation, since in this case a fixed node is indeed present in the interior of the stellar envelope, and the substitution of a fixed core at this point exactly duplicates the actual situation. The overtone

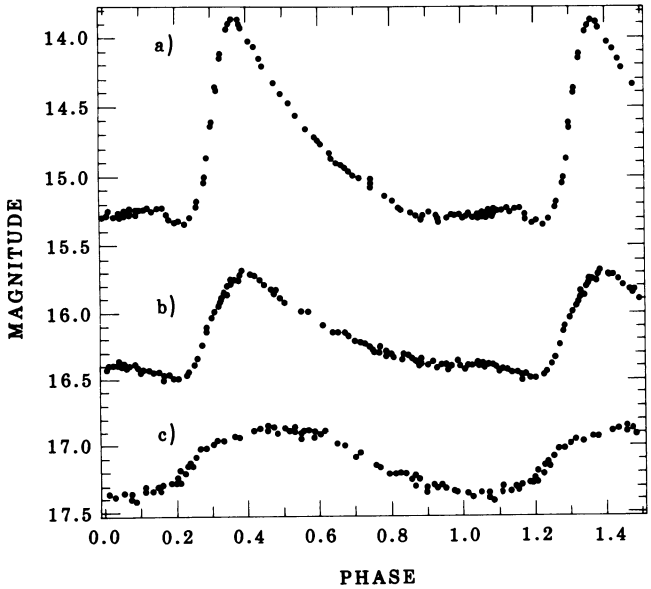

FIG. 1.—Typical light curves of RR Lyrae variables; Bailey's types a, b, and c; adopted from Ledoux and Walraven (1958). Type b and c curves are shifted by 1.6 and 2.4 mag, respectively.

model differs from the fundamental in two respects: (1) the pulsating shell is thinner, and (2) the variation of the luminosity at the base of the shell is exactly 180° out of phase with that of the fundamental case, due to the presence of the pulsation node.

The purpose of this *Letter* is to demonstrate that such a model does indeed reproduce the essential features of the fundamental and overtone modes, including the light curve shape.

II. DESCRIPTION OF THE MODEL

The derivation of the nonlinear nonadiabatic one-zone model is given in S72 and further discussed in Stellingwerf and Donohoe (1987). The equations listed here differ from the S72 model only in the choice of time scale. Here we take the dynamic time scale as the unit of time to allow direct integration of the equations without the need of iterative improvement. The resulting light curves differ slightly from previously published results: the phase relations are determined naturally by the oscillation, but the curves are not periodic, since the shell exhibits either damped or growing oscillations. A detailed derivation of the system of equations will be given in Stellingwerf and Gautschy (1987).

Taking the unit of time to be

$$\tau = \left(GM/R^3\right)^{-1/2}, \tag{1}$$

the equation of motion of the shell radius is

$$\frac{d^2X}{d\tau^2} = \frac{h}{X^q} - \frac{1}{X^2}, \tag{2}$$

where $X = R/R_0$, $q = m\Gamma_1 - 2$, Γ_1 is the adiabatic exponent of the density variation (equal to 5/3 for a simple, perfect gas), h represents the nonadiabatic portion of the pressure variation, defined by:

$$\frac{P}{P_0} = h\left(\frac{\rho}{\rho_0}\right)^{\Gamma_1}, \tag{3}$$

and m is the exponent of the density variation:

$$\frac{\rho}{\rho_0} = X^{-m}. \tag{4}$$

The value of m is related to the thickness of the shell. If r_c is the radius of the fixed core, and r_0 is the static radius of the star, then define $\eta = r_c/r_0$, and the equilibrium value for m is

$$m_0 = \frac{3}{(1-\eta^3)}. \tag{5}$$

In these models, m varies during the pulsation—its exact form is given by equation (6) of S72.

The energy equation is written

$$\frac{dh}{d\tau} = -\zeta X^{m(\Gamma_1-1)}\left[\frac{L}{L_0} - \frac{L_i}{L_0}\right], \tag{6}$$

where ζ = (dynamic time)/(thermal time) is the "nonadiabaticity parameter"—and should be near unity in the instability strip. The luminosity variation at the top and bottom of the shell are given by

$$\text{outer luminosity:} \quad \frac{L}{L_0} = X^b h^{(s+4)}, \tag{7}$$

$$\text{inner luminosity:} \quad \frac{L_i}{L_0} = X^u. \tag{8}$$

The outer luminosity variation is derived from the usual diffusion formula (see S72). The parameter u is negative for a deep damping region (opposite sign convention to that used in S72) to emphasize the symmetry of equations (7) and (8). Finally, we have

$$b = 4 + m[n - (s+4)(\Gamma_1 - 1)], \tag{9}$$

where n and s are the density and temperature exponents of the opacity in the shell (normally taken as 1 and 3, respectively).

Linear analysis shows that the shell is unstable if $b - u > 0$, and pulsationally stable otherwise. The parameter u is used to simulate the modulation of the luminosity in layers deeper than the pulsating shell, and is normally negative, since these layers damp the pulsation. Note that this enhances the driving in the shell. Taking $\Gamma_1 = 1.1$ simulates a shell with strong ionization zones, and ensures pulsational instability.

III. COMPUTED LIGHT CURVES

A series of models have been computed with the set of parameters: $\zeta = 1.0$, $\Gamma_1 = 1.1$, $n = 1$, and $s = 3$, appropriate

TABLE 1

Model Parameters

Parameter[a]	Fundamental	Fundamental–Low Amplitude	Overtone	Overtone–High Amplitude	Second Overtone
Maximum radius	1.20	1.10	1.05	1.10	1.05
m_0	10.00	10.00	15.00	15.00	20.00
u	−2.00	−2.00	2.00	2.00	1.00
Period...............	2.4600	2.4320	1.8200	1.9690	1.5620
Skewness	6.1430	3.0000	1.9410	2.5710	3.1670
Acuteness...........	2.7040	2.2260	0.4925	0.2500	0.4286
σ	0.0032	0.0087	0.0000	0.0018	0.0000
Amplitude	0.8996	0.5903	0.4030	0.7666	0.5529
H1	0.3410	0.2613	0.1800	0.2741	0.2042
R21	0.5041	0.4107	0.3498	0.5298	0.5153
R31	0.3028	0.2054	0.1246	0.3068	0.2318
R41	0.1928	0.1078	0.0450	0.1753	0.1425
R51	0.1170	0.0597	0.0161	0.1023	0.0163
Phi1	2.0550	1.9280	1.2960	1.0610	4.0700
Phi2	4.5230	4.4800	5.1330	4.8810	4.0220
Phi3	1.1970	1.1390	2.4500	2.1800	3.9560
Phi4	4.2290	4.2240	6.1500	5.7770	3.9840
Phi5	1.0770	1.1100	3.6340	3.1300	0.9535
Phi21	0.4127	0.6231	2.5420	2.7580	2.1650
Phi31	1.3150	1.6370	4.8460	5.2800	4.3110
Phi41	2.2920	2.7930	0.9672	1.5320	0.2680
Phi51	3.3680	4.0350	3.4390	4.1070	5.7340

[a] See text for explanation.

to an instability strip pulsator. Five of these models that are good representations of RR Lyrae stars will be discussed. Their parameters are shown in Table 1. Runge-Kutta integrations of the first three, in the same format as Figure 1, are shown in Figure 2. Each involves an ad hoc choice of m, a guess at u, and about two tries of X_0 to obtain reasonable luminosity amplitudes. The initial value of h is chosen to provide a smooth start, usually about 0.9. The agreement of the type a light curves with observations was already noted in S72. Here we further note the ability of this model to reproduce the type b and, especially, the type c light curves.

The fundamental mode model (curve marked "a" in Fig. 2), has radius amplitude of 20%, a shell thickness (eq. [5]) of 11%, and a value of u representative of a deep radiative damping layer (see Stellingwerf and Donohoe 1987 for a discussion of reasonable values of u). The type b model is exactly the same but is run at a lower amplitude of 10%.

The "overtone" (type c) model differs from those above in that the shell is somewhat thinner (7.2% of the radius), but more importantly, the sign of u has been changed to reflect the effect of the node: the deeper layers are still damping, but their movement is out of phase with that of the surface layers. It is clear from equation (11) that another effect of this phase change is a reduction in the driving—one cause of the smaller observed amplitudes of the type c variables.

Since this model does not include the dynamical effect of the deep damping region, a driving shell will always grow in amplitude. Figure 3 shows actual integrations for the low-

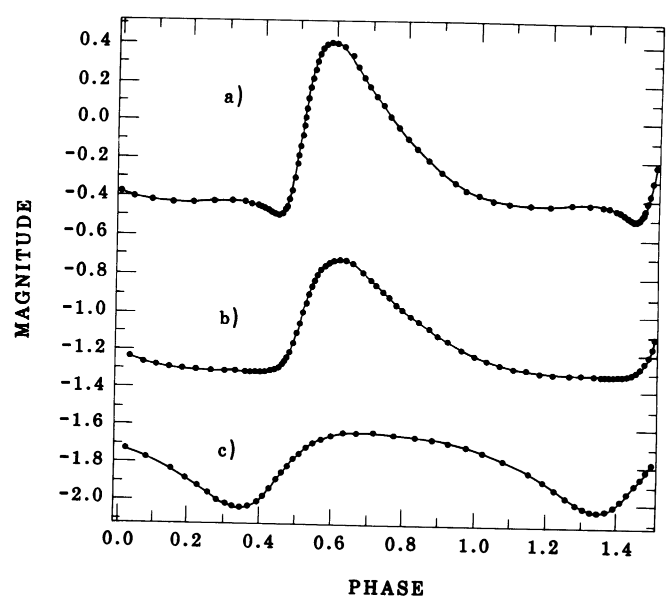

Fig. 2.—Magnitude vs. phase for the first period of the first three models shown in Table 1: (*points*), integration steps; (*lines*), Fourier fit.

amplitude fundamental and the overtone cases. Here luminosity is plotted versus time, showing the actual periods of the two modes. The period ratio is 0.740—a reasonable value. During the rapid growth of the pulsation, the development of the feature near minimum light in the fundamental is clearly

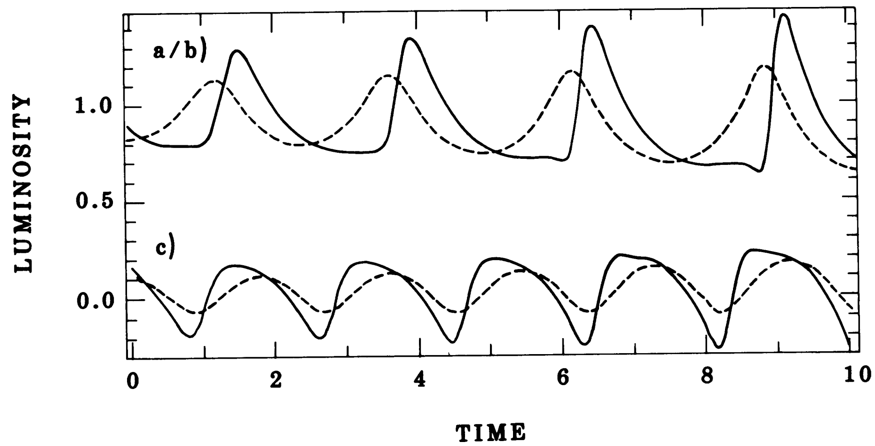

FIG. 3.—Luminosity vs. time for the low-amplitude fundamental (*top*) and the overtone (*bottom*) cases. The solid lines represent the luminosity variation at the surface; the dashed lines represent the luminosity variation at the base of the shell.

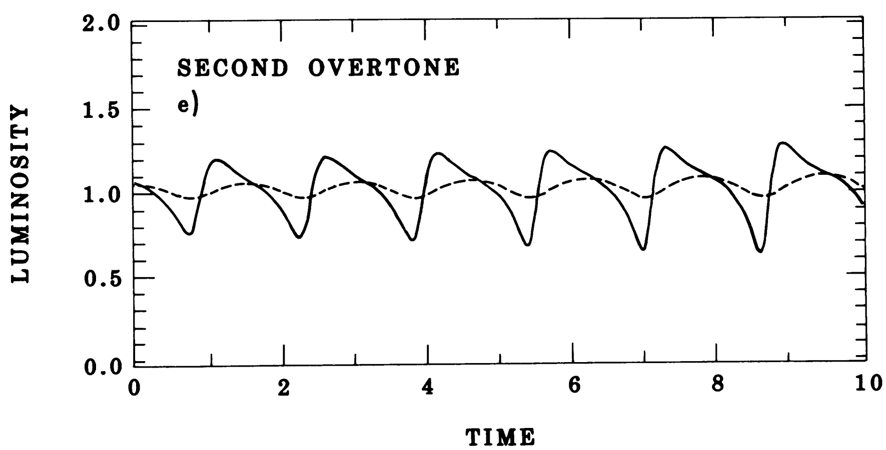

FIG. 4.—Same as Fig. 3, but for the second-overtone model

seen. The appearance of a feature near maximum light in the overtone is also apparent. Both are commonly observed in stars. An important point is that the overtone at large amplitude shows no resemblance to the fundamental. For comparison, the parameters of a large-amplitude overtone, similar to that at the end of the integration shown in Figure 3, is given in the fourth column of Table 1.

Also shown in Figure 3 is the variation of the interior luminosity. For the fundamental, L_i peaks at the phase of minimum radius; for the overtone, it *dips* at this phase. In both cases the peak in the outer luminosity lags after minimum radius—the famous "90° phase lag." The shape of the two light curves during the falling branch is strongly influenced by the variation of the interior luminosity—curving sharply downward for the fundamental, and roundly upward for the overtone. This accounts for the distinct difference in shape in the two cases. Physically, the difference of the luminosities represents a heating or cooling in the shell (eq. [9]) and will affect the dynamics of the pulsation through the h factor in equation (3). The magnitude of the difference is thus limited by pressure effects, and the two luminosities tend to track. The exception is near the phase of minimum radius, when the opacity peaks sharply and "dams up" the escaping energy, causing the observed phase delay.

This first-overtone model is so successful that we cannot resist trying a higher mode. What would a second-overtone look like? The shell is likely to be slightly thinner than the first overtone, since we now have two nodes—we take 5.3% (m = 20) as a guess. The phase reversal at the base of the pulsating shell is present, but not as effective as in the case of the first overtone, since the second, deeper node tends to wash out this effect—we take $u = 1$. Finally, we guess that the amplitude will be similar to that of the first overtone. The resulting integration is shown in Figure 4, on the same scale as Figure 3. Since the designation "type d" has already been taken to indicate a mixed-mode star, we suggest a label of "type e" RR Lyraes for any stars that may fit this category. The predicted light variation shows a much sharper peak at maximum light than the first-overtone pulsators, and this may be a valuable clue to identification.

Type "e" stars are expected (from stability analyses) to have the shortest periods, the hottest effective temperatures, and perhaps the lowest luminosities of any group of observed RR Lyraes. Candidates for type "e" stars include variable 20 in NGC 4833 (Demers and Wehlau 1977) and variables 56 and 97 in IC 4499 (Clement, Dickens, and Bingham 1979). Since the amplitude of our model was chosen arbitrarily, we cannot exclude low-amplitude sinusoidal pulsations as type "e" stars.

IV. FOURIER PARAMETERS

The parameters of a standard Fourier fit to the light curves are also given in Table 1; see Stellingwerf and Donohoe (1986, 1987) for a detailed definition of these parameters. The skewness has large values for short rising branches; the acuteness has large values for narrow peaks; and σ is the standard deviation of the fit. Amplitude is the total amplitude, H1 is the amplitude of the lowest harmonic of the fit, and RI1 is the ratio of the Ith amplitude and the first. PhiI is the phase of the Ith harmonic, and PhiI1 is the phase relative to the first.

It is beyond the scope of this *Letter* to fully analyze this data, and the intention is to simply present the numbers for future reference, with a few, brief comments. See Simon and Teays (1982), Petersen (1984), or Stellingwerf and Dickens (1987) for a review of observed parameters.

The parameters of the fundamental model agree with those of observed stars. The first overtone has an acuteness somewhat lower than observed, indicating that the computed curve is too "fat," caused perhaps by an incorrect value for u. The Fourier parameters fall on an extrapolation of the observed relation. The second-overtone model has an even lower value of the acuteness, but a higher value of the skewness. The Fourier parameters are also unusual—a hybrid of overtone and fundamental values. This should serve as a test for suspected second overtones.

V. CONCLUSION

We have demonstrated that a very simple one-zone model is sufficient to compute the light variation of the three observed types of RR Lyrae stars and have thus explained the unique shape of the type c light curve as a consequence of mode of pulsation. We then use this model to predict the shape of the second-overtone light curve and dub these undiscovered stars "type e" RR Lyraes.

In some sense, these results are quite general and are not restricted to RR Lyraes. Any radial pulsator should show these effects, although some adjustment of parameters may be necessary: for luminous pulsators or Beta Cepheids, radiation pressure is important, while for cool variables convection will play a leading role.

This research was supported in part by National Science Foundation grant AST84-11029, through Mission Research Corporation.

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